EPIMODELS

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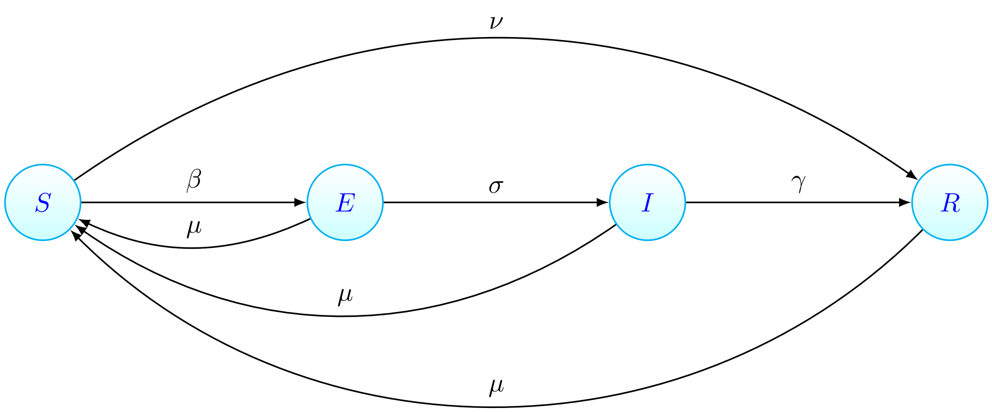
April 5, 2020

1 SEIR model



Consider the model defined by the following system of ordinary differential equations:

$\frac{dS}{dt} = \mu(N - S) - \beta\frac{SI}{N} - \nu S \ , \ S(t_0) = S_0$

$\frac{dE}{dt} = \beta\frac{SI}{N} - (\mu + \sigma)E \ , \ E(t_0) = E_0$

$\frac{dI}{dt} = \sigma E - (\mu + \gamma)I \ , \ I(t_0) = I_0$

$\frac{dR}{dt} = \gamma I - \mu R + \nu S \ , \ R(t_0) = R_0$

The parameter β characterizes the speed of contagion, at which susceptible individuals become exposed. The parameter σ is the constant rate at which exposed individuals become infected. The parameter γ is a constant rate at which infected individuals recover. The parameter μ is the constant natural mortality rate unrelated to the disease being modelled. The model is formulated under the constant population assumption ($S+E+I+R = N = Population$), so that the natural mortality (μ) is counterbalanced by the equivalent fertility, refreshing the susceptible population.

The vaccination rate (ν) is transferring individuals from susceptible state to recovered (assuming resistance to the disease).

We can simplify the model by assuming away these two effects (letting both μ and ν equal to zero). Under this assumption the model becomes:

$$\frac{dS}{dt} = -\beta \frac{SI}{N} \ , \ S(t_0) = S_0$$

$$\frac{dE}{dt} = \beta \frac{SI}{N} - \sigma E \ , \ E(t_0) = E_0$$

$$\frac{dI}{dt} = \sigma E - \gamma I \ , \ I(t_0) = I_0$$

$$\frac{dR}{dt} = \gamma I \ , \ R(t_0) = R_0$$

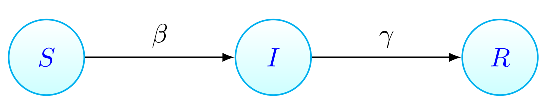
Note that this effectively becomes a SIR model if we further assume parameter σ to be equal to zero and combine the exposed and infected states together:

$$\frac{dS}{dt} = -\beta \frac{SI}{N} \ , \ S(t_0) = S_0$$

$$\frac{dI}{dt} = \beta \frac{SI}{N} - \gamma I \ , \ I(t_0) = I_0$$

$$\frac{dR}{dt} = \gamma I \ , \ R(t_0) = R_0$$

2 SIR model



The SIR model is also formulated under the constant population assumption $Population = N = S + I + R$ and is characterized by the following system of differential equations:

$$\frac{dS}{dt} = -\beta \frac{SI}{N} \ , \ S(t_0) = S_0$$

$$\frac{dI}{dt} = \beta \frac{SI}{N} - \gamma I \ , \ I(t_0) = I_0$$

$$\frac{dR}{dt} = \gamma I \ , \ R(t_0) = R_0$$