1 SEIR model

Consider the model defined by the following system of ordinary differential equations:

\[
\begin{align*}
\frac{dS}{dt} &= \mu(N - S) - \beta \frac{SI}{N} - \nu S, \quad S(t_0) = S_0 \\
\frac{dE}{dt} &= \beta \frac{SI}{N} - (\mu + \sigma)E, \quad E(t_0) = E_0 \\
\frac{dI}{dt} &= \sigma E - (\mu + \gamma)I, \quad I(t_0) = I_0 \\
\frac{dR}{dt} &= \gamma I - \mu R + \nu S, \quad R(t_0) = R_0
\end{align*}
\]
The parameter $\beta$ characterizes the speed of contagion, at which susceptible individuals become exposed. The parameter $\sigma$ is the constant rate at which exposed individuals become infected. The parameter $\gamma$ is a constant rate at which infected individuals recover. The parameter $\mu$ is the constant natural mortality rate unrelated to the disease being modelled. The model is formulated under the constant population assumption ($S + E + I + R = N = \text{Population}$), so that the natural mortality ($\mu$) is counterbalanced by the equivalent fertility, refreshing the susceptible population.

The vaccination rate ($\nu$) is transferring individuals from susceptible state to recovered (assuming resistance to the disease).

We can simplify the model by assuming away these two effects (letting both $\mu$ and $\nu$ equal to zero). Under this assumption the model becomes:

\[
\begin{align*}
\frac{dS}{dt} &= -\beta \frac{SI}{N}, \quad S(t_0) = S_0 \\
\frac{dE}{dt} &= \beta \frac{SI}{N} - \sigma E, \quad E(t_0) = E_0 \\
\frac{dI}{dt} &= \sigma E - \gamma I, \quad I(t_0) = I_0 \\
\frac{dR}{dt} &= \gamma I, \quad R(t_0) = R_0
\end{align*}
\]

Note that this effectively becomes a SIR model if we further assume parameter $\sigma$ to be equal to zero and combine the exposed and infected states together:

\[
\begin{align*}
\frac{dS}{dt} &= -\beta \frac{SI}{N}, \quad S(t_0) = S_0 \\
\frac{dI}{dt} &= \beta \frac{SI}{N} - \gamma I, \quad I(t_0) = I_0 \\
\frac{dR}{dt} &= \gamma I, \quad R(t_0) = R_0
\end{align*}
\]

2 SIR model

The SIR model is also formulated under the constant population assumption $\text{Population} = N = S + I + R$ and is characterized by the following system of
differential equations:

\[ \frac{dS}{dt} = -\beta \frac{SI}{N} , \quad S(t_0) = S_0 \]

\[ \frac{dI}{dt} = \beta \frac{SI}{N} - \gamma I , \quad I(t_0) = I_0 \]

\[ \frac{dR}{dt} = \gamma I , \quad R(t_0) = R_0 \]