COVID-19
Modelling the Epidemic

The SIR Model

Sergiy Radyakin

Paolo Verme

World Bank

Definitions – Stocks of people

- S = Number of **susceptible** persons. This is the number of people that can potentially become infected. With a new virus and at time 0 of an epidemic, this is usually the entire population
- I = Number of **infected** persons. Number of people who contracted the infection and are infectious. They can potentially transmit the virus to others
- R = Number of **recovered** people. Those who have been infected (I) and have recovered
- N = **Population**

Definitions – Shares of population

- s = S/N = Share of the population that is **susceptible** to the virus
- i = I/N = share of the population that is **infected**
- r = R/N = share of the population that has **recovered**

Assumptions

$$\mathbf{S + I + R = N = Population}$$

$$\mathbf{s + i + r = 1}$$

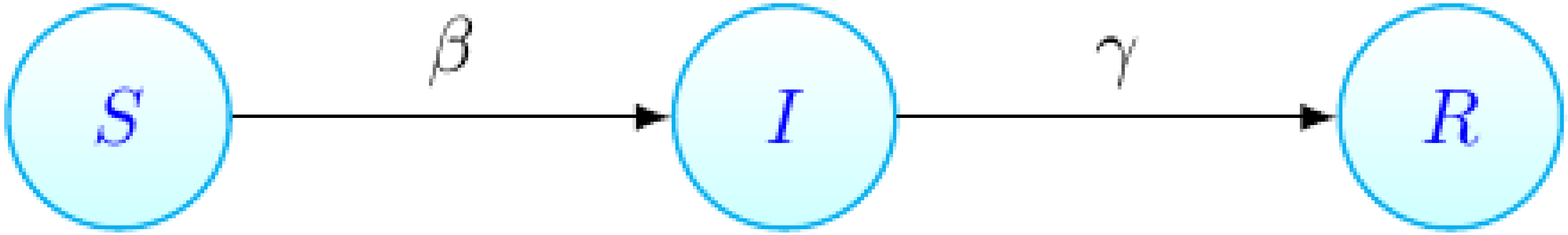
$\Rightarrow$ All persons of the a population can be assigned to one of these three categories at any point of the epidemic

$\Rightarrow$ Once recovered, a person cannot become infected again (this person becomes immune)

$\Rightarrow$ Natural births and natural deaths of the population are ignored

$\Rightarrow$ International migration is ignored

The SIR Model



Parameter β

β = number of contacts per day per infected individual

=1/average period an infectious person makes an infecting contact (measured in days)

<u>Example:</u>

<u>1/2 means that an infected person makes an infecting contact every two days</u>

The average period an infectious person makes an infecting contact is exogenous and established by the researcher based on knowledge of the disease.

Parameter γ

γ = R/I fraction of the infectious group that will recover

= 1/average period of infectiousness (measured in days)

<u>Example:</u>

<u>1/3 means that 1/3 of the infectious group will recover each day</u>

The average period of infectiousness is exogenous and established by the researcher based on knowledge of the disease.

System of Differential Equations

With three variables (s, i, r) and two parameters (β, γ), the SIR model is represented by a system of three equations.

To plot s, i and r over time, you need to differentiate these three variables with respect to time (ds/dt; di/dt; dr/dt). This gives you three differential equations.

Note that these equations are nonlinear.

$$\frac{ds}{dt} = -\beta s(t)i(t)$$

$$\frac{di}{dt} = \beta s(t)i(t) - \gamma i(t)$$

$$\frac{dr}{dt} = \gamma i(t)$$

Solving a system of Differential Equations

There are many ways to solve a system of differential equations including numerical and non-numerical methods.

One simple way is to derive formulas with the Euler method.

We can use these formulas to estimate the value of each variable (s, i, r) in each point in time (t) based on the parameters chosen (β, γ).

These formulas are easily coded in any programming language and can be used to plot the s, i and r curves.

$$s(t) = s(t-1) - \beta s(t-1) i(t-1)$$

$$\mathrm{i}(t) = i(t-1) + \left(\beta s(t-1) i(t-1) - \gamma i(t-1)\right)$$

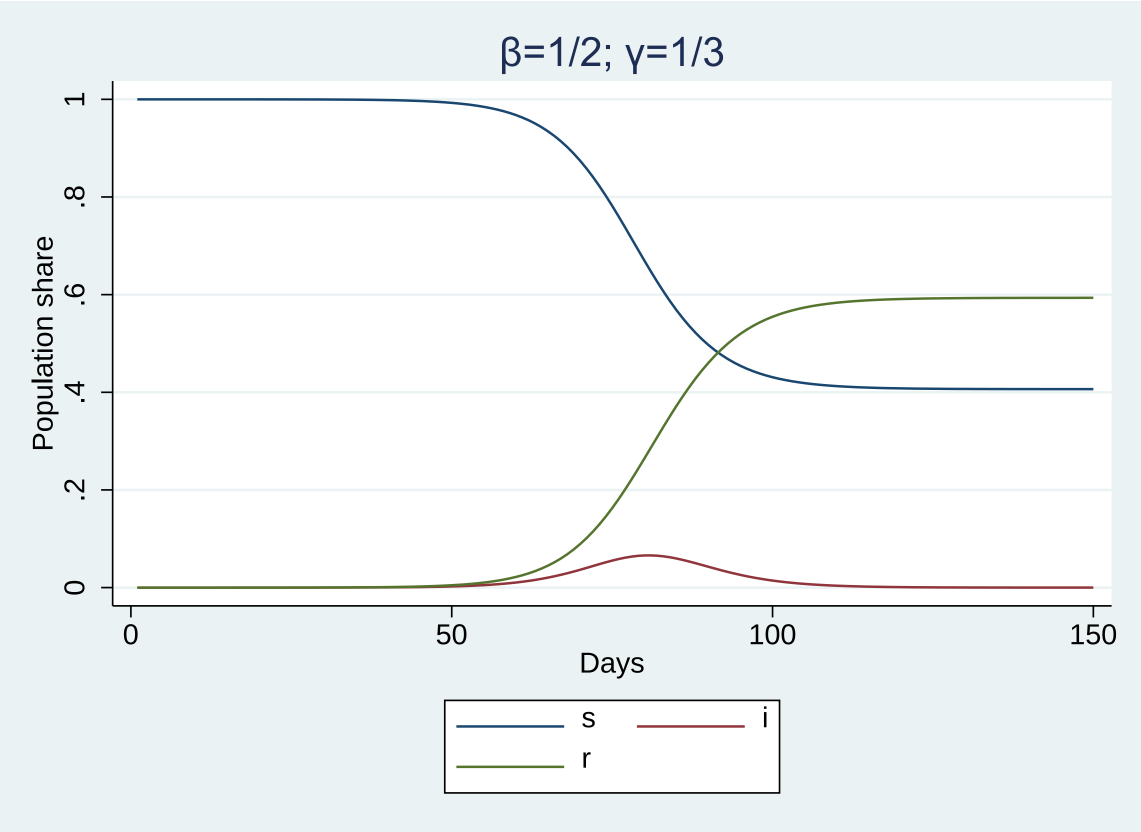
$$\mathrm{r}(t) = r(t-1) - \gamma i(t-1)$$

With Δ(t)=1

The SIR curves plotted using STATA

Since the population is equal to 1, when the infection rate increases, the recovery rate increases and the susceptible rate decreases.

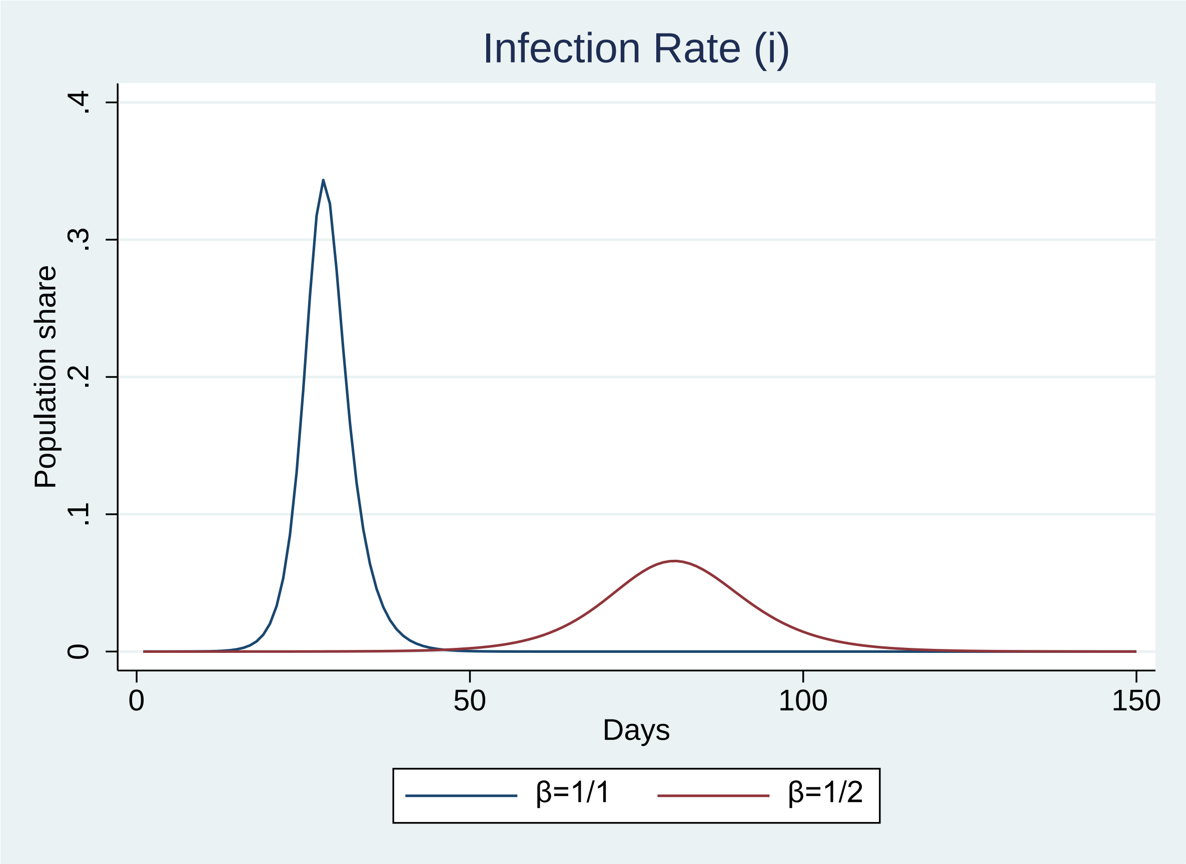
The rate of increase and decrease of these parameters depend on the parameters β and γ.



What happens when β varies?

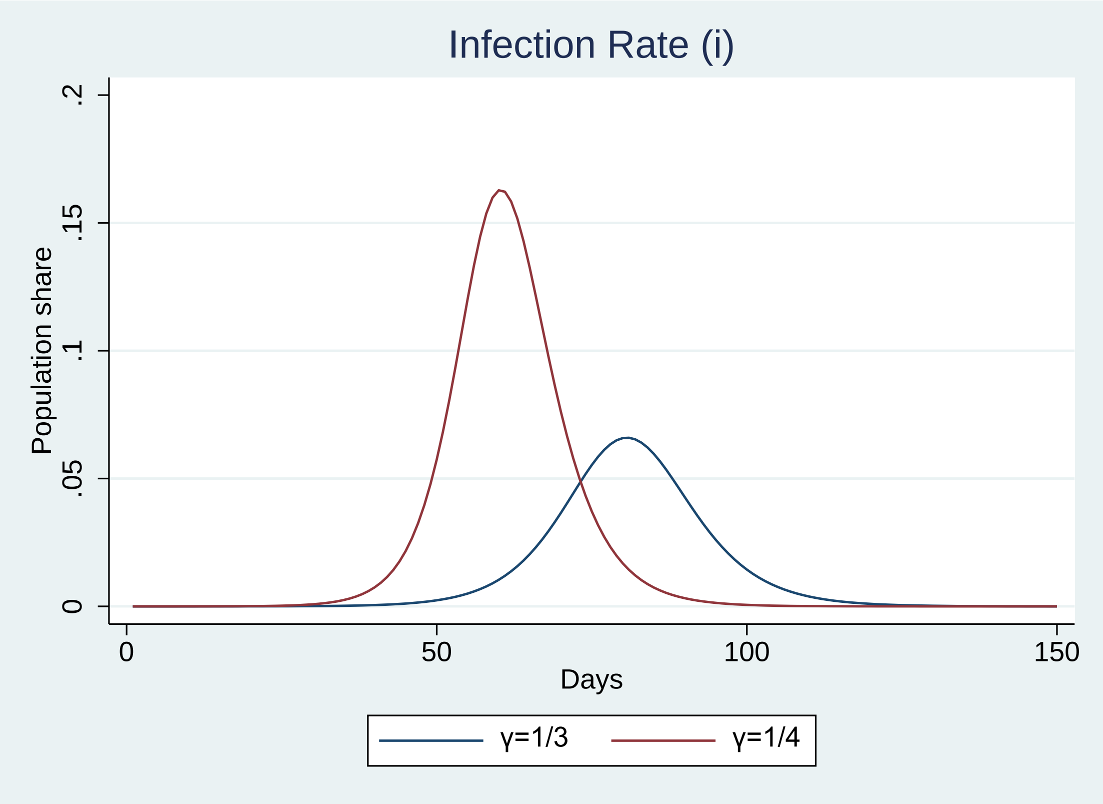
If the **average period an infectious person makes an infecting contact** is increased, β decreases, infections per day decrease and the infection rate curve is "flattened".

<u>This is why **social distancing** works. It reduces the frequencies of the contacts between infectious and susceptible people (β).</u>



What happens when γ varies?

If the **average period of infectiousness increases**, γ decreases, infections per day increase and the infection rate curve rises.



<u>This is why **treatment** may help. It can reduce the average period of infectiousness.</u>

References:

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